

Fast simultaneous approximation of sine and cosine

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implementation

```
iterations = 4;
w = radians / iterations;

float sc = 1.f - w * w;   float ss = w + w * sc;           //    two iterations
sc -= w * ss;   ss += w * sc;
sc -= w * ss;   ss += w * sc;
sc -= .5f * w * ss;           //    average with next iteration

# of multiplications = iterations * 2 for both sine and cosine output
tolerance = +/- pi (radians)
```

Accuracy generally improves with smaller angles and more iterations. Suitability for nonrecurring and noncritical operations eg. effects.

method

The "fast sine" is a dsp perennial referenced by several names (imo quadrature oscillator seems most descriptive).

```
w = 2.f * pi * freq_in_hertz / samplerate;           //    angular frequency
sc = 1.f;   ss = 0.f;                               //    initialise

sc -= w * ss;   ss += w * sc;                       //    loop
*sine_out = ss;
```

In practical use, floating point error requires the state variables to be occasionally reset to magnitude = 1 eg. multiply both by $1.f / \sqrt{sc * sc + ss * ss}$

Obviously we have the cartesian coordinates of a rotating unit vector, so can produce sine and cosine and increase accuracy by "oversampling" the operation by repeating it with an appropriately scaled frequency coefficient. This allows fast derivation of transcendentals for applications where low fidelity suffices eg. particle based explosion effects.

accuracy

Check the charts for the remaining pages.

license

Since there's no way to legate usage once the method is shared, consideration of licensing is somewhat vile even as an excuse to write more stuff. Algorithm published under the yall can't copyright no beats flava license and I'm totally serious I've been using this dsp for years but only considered it "as an approximation method" yesterday and normally I don't do this a whole lot but now through november if this saves a bunch of cycles off your project and you are so happy send me a tenner or something, appreciated great :)

angle: 1 * pi / 16

iterations: 1 error cos: 6.556510925e-07 error sin: 4.068389535e-05
iterations: 2 error cos: 1.788139343e-07 error sin: 1.016631722e-05
iterations: 3 error cos: 5.960464478e-08 error sin: 4.522502422e-06
iterations: 4 error cos: 5.960464478e-08 error sin: 2.540647984e-06
iterations: 5 error cos: 5.960464478e-08 error sin: 1.631677151e-06
iterations: 6 error cos: 5.960464478e-08 error sin: 1.132488251e-06
iterations: 7 error cos: 0 error sin: 8.344650269e-07
iterations: 8 error cos: 0 error sin: 6.332993507e-07
iterations: 9 error cos: 0 error sin: 5.103647709e-07
iterations: 10 error cos: 0 error sin: 4.135072231e-07
iterations: 11 error cos: 5.960464478e-08 error sin: 3.352761269e-07
iterations: 12 error cos: 0 error sin: 2.86847353e-07
iterations: 13 error cos: 5.960464478e-08 error sin: 2.421438694e-07
iterations: 14 error cos: 5.960464478e-08 error sin: 2.12341547e-07
iterations: 15 error cos: 5.960464478e-08 error sin: 1.825392246e-07
iterations: 16 error cos: 1.192092896e-07 error sin: 1.527369022e-07

angle: 7 * pi / 16

iterations: 1 error cos: 0.001516819 error sin: 0.01382374763
iterations: 2 error cos: 0.0003719329834 error sin: 0.003356218338
iterations: 3 error cos: 0.0001647472382 error sin: 0.001483738422
iterations: 4 error cos: 9.256601334e-05 error sin: 0.0008330643177
iterations: 5 error cos: 5.924701691e-05 error sin: 0.0005327165127
iterations: 6 error cos: 4.106760025e-05 error sin: 0.0003697574139
iterations: 7 error cos: 3.02195549e-05 error sin: 0.0002716183662
iterations: 8 error cos: 2.312660217e-05 error sin: 0.000207901001
iterations: 9 error cos: 1.82390213e-05 error sin: 0.000164270401
iterations: 10 error cos: 1.484156556e-05 error sin: 0.000133007648
iterations: 11 error cos: 1.221895218e-05 error sin: 0.0001099407673
iterations: 12 error cos: 1.02519989e-05 error sin: 9.241700172e-05
iterations: 13 error cos: 8.881092072e-06 error sin: 7.86781311e-05
iterations: 14 error cos: 7.510185242e-06 error sin: 6.785988808e-05
iterations: 15 error cos: 6.496906281e-06 error sin: 5.912780762e-05
iterations: 16 error cos: 5.841255188e-06 error sin: 5.194544792e-05

angle: 2 * pi / 16

iterations: 1 error cos: 1.019239426e-05 error sin: 0.0003252699971
iterations: 2 error cos: 2.562999725e-06 error sin: 8.112937212e-05
iterations: 3 error cos: 1.192092896e-06 error sin: 3.604590893e-05
iterations: 4 error cos: 6.556510925e-07 error sin: 2.02730298e-05
iterations: 5 error cos: 4.768371582e-07 error sin: 1.297891144e-05
iterations: 6 error cos: 3.576278687e-07 error sin: 9.015202522e-06
iterations: 7 error cos: 2.384185791e-07 error sin: 6.623566151e-06
iterations: 8 error cos: 1.788139343e-07 error sin: 5.066394806e-06
iterations: 9 error cos: 5.960464478e-08 error sin: 4.008412361e-06
iterations: 10 error cos: 5.960464478e-08 error sin: 3.255903721e-06
iterations: 11 error cos: 1.192092896e-07 error sin: 2.689659595e-06
iterations: 12 error cos: 5.960464478e-08 error sin: 2.257525921e-06
iterations: 13 error cos: 5.960464478e-08 error sin: 1.92249794e-06
iterations: 14 error cos: 0 error sin: 1.668930054e-06
iterations: 15 error cos: 5.960464478e-08 error sin: 1.445412636e-06
iterations: 16 error cos: 1.192092896e-07 error sin: 1.259148121e-06

angle: 8 * pi / 16

iterations: 1 error cos: 0.002582550049 error sin: 0.02057445049
iterations: 2 error cos: 0.0006294250488 error sin: 0.004949450493
iterations: 3 error cos: 0.0002784729004 error sin: 0.002184540033
iterations: 4 error cos: 0.0001564025879 error sin: 0.001225829124
iterations: 5 error cos: 0.0001000165939 error sin: 0.0007836520672
iterations: 6 error cos: 6.943941116e-05 error sin: 0.0005438923836
iterations: 7 error cos: 5.090236664e-05 error sin: 0.0003994703293
iterations: 8 error cos: 3.904104233e-05 error sin: 0.000305718277
iterations: 9 error cos: 3.087520599e-05 error sin: 0.0002415180206
iterations: 10 error cos: 2.497434616e-05 error sin: 0.0001956224442
iterations: 11 error cos: 2.074241638e-05 error sin: 0.0001616477966
iterations: 12 error cos: 1.734495163e-05 error sin: 0.0001358687878
iterations: 13 error cos: 1.484156556e-05 error sin: 0.0001157820225
iterations: 14 error cos: 1.269578934e-05 error sin: 9.983778e-05
iterations: 15 error cos: 1.096725464e-05 error sin: 8.699297905e-05
iterations: 16 error cos: 9.715557098e-06 error sin: 7.638335228e-05

angle: 3 * pi / 16

iterations: 1 error cos: 5.143880844e-05 error sin: 0.001096710563
iterations: 2 error cos: 1.28149983e-05 error sin: 0.0002727359533
iterations: 3 error cos: 5.662441254e-06 error sin: 0.000121101737
iterations: 4 error cos: 3.218650818e-06 error sin: 6.809830666e-05
iterations: 5 error cos: 2.026557922e-06 error sin: 4.358589649e-05
iterations: 6 error cos: 1.37090683e-06 error sin: 3.026425838e-05
iterations: 7 error cos: 1.072883606e-06 error sin: 2.224743366e-05
iterations: 8 error cos: 8.344650269e-07 error sin: 1.703202724e-05
iterations: 9 error cos: 7.152557373e-07 error sin: 1.345574856e-05
iterations: 10 error cos: 4.768371582e-07 error sin: 1.092255116e-05
iterations: 11 error cos: 4.768371582e-07 error sin: 9.000301361e-06
iterations: 12 error cos: 2.980232239e-07 error sin: 7.584691048e-06
iterations: 13 error cos: 2.980232239e-07 error sin: 6.467103958e-06
iterations: 14 error cos: 2.980232239e-07 error sin: 5.558133125e-06
iterations: 15 error cos: 1.788139343e-07 error sin: 4.87267971e-06
iterations: 16 error cos: 2.384185791e-07 error sin: 4.261732101e-06

angle: 9 * pi / 16

iterations: 1 error cos: 0.004127621651 error sin: 0.02919733524
iterations: 2 error cos: 0.0009990930557 error sin: 0.00695002079
iterations: 3 error cos: 0.0004414319992 error sin: 0.003061890602
iterations: 4 error cos: 0.000248411313 error sin: 0.001717031002
iterations: 5 error cos: 0.0001584291458 error sin: 0.001097381115
iterations: 6 error cos: 0.000109910965 error sin: 0.0007615089417
iterations: 7 error cos: 8.076429367e-05 error sin: 0.0005592107773
iterations: 8 error cos: 6.186962128e-05 error sin: 0.0004280209541
iterations: 9 error cos: 4.887580872e-05 error sin: 0.0003381371498
iterations: 10 error cos: 3.951787949e-05 error sin: 0.0002738237381
iterations: 11 error cos: 3.272294998e-05 error sin: 0.0002263188362
iterations: 12 error cos: 2.747771424e-05 error sin: 0.0001901388166
iterations: 13 error cos: 2.34246254e-05 error sin: 0.0001619458199
iterations: 14 error cos: 2.020597458e-05 error sin: 0.0001397132874
iterations: 15 error cos: 1.758337021e-05 error sin: 0.00012165308
iterations: 16 error cos: 1.5437603e-05 error sin: 0.0001069903374

angle: 4 * pi / 16

iterations: 1 error cos: 0.000162422657 error sin: 0.0025960356
iterations: 2 error cos: 4.035234451e-05 error sin: 0.0006429105997
iterations: 3 error cos: 1.788139343e-05 error sin: 0.0002852529827
iterations: 4 error cos: 1.007318497e-05 error sin: 0.0001603513956
iterations: 5 error cos: 6.437301636e-06 error sin: 0.0001025795937
iterations: 6 error cos: 4.470348358e-06 error sin: 7.124245167e-05
iterations: 7 error cos: 3.278255463e-06 error sin: 5.234777927e-05
iterations: 8 error cos: 2.503395081e-06 error sin: 4.006922245e-05
iterations: 9 error cos: 1.966953278e-06 error sin: 3.165006638e-05
iterations: 10 error cos: 1.668930054e-06 error sin: 2.562999725e-05
iterations: 11 error cos: 1.311302185e-06 error sin: 2.118945122e-05
iterations: 12 error cos: 1.132488251e-06 error sin: 1.780688763e-05
iterations: 13 error cos: 1.013278961e-06 error sin: 1.518428326e-05
iterations: 14 error cos: 8.344650269e-07 error sin: 1.306831837e-05
iterations: 15 error cos: 6.556510925e-07 error sin: 1.141428947e-05
iterations: 16 error cos: 5.960464478e-07 error sin: 1.002848148e-05

angle: 10 * pi / 16

iterations: 1 error cos: 0.006275594234 error sin: 0.03990274668
iterations: 2 error cos: 0.001507222652 error sin: 0.009385168552
iterations: 3 error cos: 0.0006651282831 error sin: 0.004126191139
iterations: 4 error cos: 0.000371250763 error sin: 0.002312302589
iterations: 5 error cos: 0.0002385377884 error sin: 0.001477301121
iterations: 6 error cos: 0.0001655220985 error sin: 0.001024961472
iterations: 7 error cos: 0.0001215338707 error sin: 0.0007526278496
iterations: 8 error cos: 9.3102445514e-05 error sin: 0.0005759596825
iterations: 9 error cos: 7.349252701e-05 error sin: 0.000450218582
iterations: 10 error cos: 5.960464478e-05 error sin: 0.000368535186
iterations: 11 error cos: 4.917383194e-05 error sin: 0.0003045201302
iterations: 12 error cos: 4.130601883e-05 error sin: 0.0002558231354
iterations: 13 error cos: 3.528594971e-05 error sin: 0.0002179741859
iterations: 14 error cos: 3.039836884e-05 error sin: 0.000187933445
iterations: 15 error cos: 2.652406693e-05 error sin: 0.0001637339592
iterations: 16 error cos: 2.330541611e-05 error sin: 0.0001438856125

angle: 5 * pi / 16

iterations: 1 error cos: 0.0003960728645 error sin: 0.005061477423
iterations: 2 error cos: 9.804964066e-05 error sin: 0.001246780157
iterations: 3 error cos: 4.351139069e-05 error sin: 0.000552624464
iterations: 4 error cos: 2.443790436e-05 error sin: 0.0003105401993
iterations: 5 error cos: 1.561641693e-05 error sin: 0.000198662281
iterations: 6 error cos: 1.084804535e-05 error sin: 0.000137925148
iterations: 7 error cos: 7.9870224e-06 error sin: 0.0001012980938
iterations: 8 error cos: 6.139278412e-06 error sin: 7.754564285e-05
iterations: 9 error cos: 4.827976227e-06 error sin: 6.127357483e-05
iterations: 10 error cos: 3.87430191e-06 error sin: 4.96506691e-05
iterations: 11 error cos: 3.218650818e-06 error sin: 4.103779793e-05
iterations: 12 error cos: 2.74181366e-06 error sin: 3.4481287e-05
iterations: 13 error cos: 2.324581146e-06 error sin: 2.938508987e-05
iterations: 14 error cos: 1.907348633e-06 error sin: 2.533197403e-05
iterations: 15 error cos: 1.788139343e-06 error sin: 2.205371857e-05
iterations: 16 error cos: 1.549720764e-06 error sin: 1.937150955e-05

angle: 11 * pi / 16

iterations: 1 error cos: 0.009163081646 error sin: 0.05289292336
iterations: 2 error cos: 0.002181708813 error sin: 0.01227402687
iterations: 3 error cos: 0.0009613037109 error sin: 0.005384087563
iterations: 4 error cos: 0.000539124012 error sin: 0.003014802933
iterations: 5 error cos: 0.0003445744514 error sin: 0.001925468445
iterations: 6 error cos: 0.0002391338348 error sin: 0.00133562088
iterations: 7 error cos: 0.0001756548882 error sin: 0.0009805560112
iterations: 8 error cos: 0.0001343488693 error sin: 0.0007504224777
iterations: 9 error cos: 0.000106215477 error sin: 0.0005927681923
iterations: 10 error cos: 8.600950241e-05 error sin: 0.0004799962044
iterations: 11 error cos: 7.110834122e-05 error sin: 0.00039668911
iterations: 12 error cos: 5.966424942e-05 error sin: 0.000333091736
iterations: 13 error cos: 5.090236664e-05 error sin: 0.000283969231
iterations: 14 error cos: 4.374980927e-05 error sin: 0.0002448558807
iterations: 15 error cos: 3.826618195e-05 error sin: 0.0002132058144
iterations: 16 error cos: 3.349781036e-05 error sin: 0.0001873970032

angle: 6 * pi / 16

iterations: 1 error cos: 0.0008201003075 error sin: 0.0087274611
iterations: 2 error cos: 0.0002021193504 error sin: 0.002135664225
iterations: 3 error cos: 8.95857811e-05 error sin: 0.0009454786777
iterations: 4 error cos: 5.03632019e-05 error sin: 0.0005311071873
iterations: 5 error cos: 3.218650818e-05 error sin: 0.0003396886706
iterations: 6 error cos: 2.235174179e-05 error sin: 0.0002358257771
iterations: 7 error cos: 1.639127731e-05 error sin: 0.0001732409
iterations: 8 error cos: 1.25169754e-05 error sin: 0.0001325905323
iterations: 9 error cos: 9.894371033e-06 error sin: 0.0001047849655
iterations: 10 error cos: 8.046627045e-06 error sin: 8.484721184e-05
iterations: 11 error cos: 6.675720215e-06 error sin: 7.009506226e-05
iterations: 12 error cos: 5.543231964e-06 error sin: 5.891919136e-05
iterations: 13 error cos: 4.649162292e-06 error sin: 5.01871109e-05
iterations: 14 error cos: 4.11272049e-06 error sin: 4.327297211e-05
iterations: 15 error cos: 3.63588331e-06 error sin: 3.764033318e-05
iterations: 16 error cos: 3.218650818e-06 error sin: 3.311038017e-05

angle: 12 * pi / 16

iterations: 1 error cos: 0.01293885708 error sin: 0.06836122274
iterations: 2 error cos: 0.003051161766 error sin: 0.01562684774
iterations: 3 error cos: 0.001342177391 error sin: 0.006837785244
iterations: 4 error cos: 0.0007522702217 error sin: 0.003825545311
iterations: 5 error cos: 0.0004805922508 error sin: 0.002442359924
iterations: 6 error cos: 0.000334879875 error sin: 0.001693725586
iterations: 7 error cos: 0.0002447962761 error sin: 0.00124335289
iterations: 8 error cos: 0.0001873373985 error sin: 0.0009514689445
iterations: 9 error cos: 0.0001480579376 error sin: 0.0007514953613
iterations: 10 error cos: 0.0001199245453 error sin: 0.0006085038185
iterations: 11 error cos: 9.900331497e-05 error sin: 0.0005029439926
iterations: 12 error cos: 8.33272934e-05 error sin: 0.000424772222
iterations: 13 error cos: 7.086992264e-05 error sin: 0.0003599524498
iterations: 14 error cos: 6.109476089e-05 error sin: 0.0003103613853
iterations: 15 error cos: 5.322694778e-05 error sin: 0.0002702474594
iterations: 16 error cos: 4.684925079e-05 error sin: 0.0002375245094

angle: 13 * pi / 16

iterations: 1 error cos: 0.01776367426 error sin: 0.08649134636
iterations: 2 error cos: 0.004144728184 error sin: 0.01944422722
iterations: 3 error cos: 0.001820087433 error sin: 0.008485257626
iterations: 4 error cos: 0.001019477844 error sin: 0.004742920399
iterations: 5 error cos: 0.0006512403488 error sin: 0.003026664257
iterations: 6 error cos: 0.000451862812 error sin: 0.002098560333
iterations: 7 error cos: 0.0003315806389 error sin: 0.001540362835
iterations: 8 error cos: 0.0002538561821 error sin: 0.001178622246
iterations: 9 error cos: 0.000200510025 error sin: 0.0009308457375
iterations: 10 error cos: 0.000162422657 error sin: 0.0007537603378
iterations: 11 error cos: 0.0001341700554 error sin: 0.0006227493286
iterations: 12 error cos: 0.0001128315926 error sin: 0.0005232095718
iterations: 13 error cos: 9.608268738e-05 error sin: 0.0004457831383
iterations: 14 error cos: 8.279085159e-05 error sin: 0.0003842711449
iterations: 15 error cos: 7.212162018e-05 error sin: 0.0003347992897
iterations: 16 error cos: 6.330013275e-05 error sin: 0.0002942681313

angle: 1 * pi / 147

iterations: 1 error cos: 0 error sin: 1.626089215e-06
iterations: 2 error cos: 5.960464478e-08 error sin: 4.060566425e-07
iterations: 3 error cos: 0 error sin: 1.806765795e-07
iterations: 4 error cos: 5.960464478e-08 error sin: 1.005828381e-07
iterations: 5 error cos: 5.960464478e-08 error sin: 6.332993507e-08
iterations: 6 error cos: 5.960464478e-08 error sin: 4.470348358e-08
iterations: 7 error cos: 0 error sin: 3.166496754e-08
iterations: 8 error cos: 0 error sin: 2.235174179e-08
iterations: 9 error cos: 5.960464478e-08 error sin: 2.048909664e-08
iterations: 10 error cos: 5.960464478e-08 error sin: 1.490116119e-08
iterations: 11 error cos: 0 error sin: 1.303851604e-08
iterations: 12 error cos: 5.960464478e-08 error sin: 1.11758709e-08
iterations: 13 error cos: 0 error sin: 9.313225746e-09
iterations: 14 error cos: 0 error sin: 7.450580597e-09
iterations: 15 error cos: 0 error sin: 5.587935448e-09
iterations: 16 error cos: 0 error sin: 5.587935448e-09

angle: 14 * pi / 16

iterations: 1 error cos: 0.02380937338 error sin: 0.1074565053
iterations: 2 error cos: 0.005491197109 error sin: 0.02371627092
iterations: 3 error cos: 0.002406716347 error sin: 0.01031953096
iterations: 4 error cos: 0.001347303391 error sin: 0.005762577057
iterations: 5 error cos: 0.000860334427 error sin: 0.003675639629
iterations: 6 error cos: 0.0005967617035 error sin: 0.00254791975
iterations: 7 error cos: 0.0004380941391 error sin: 0.001869916916
iterations: 8 error cos: 0.0003352761269 error sin: 0.001430630684
iterations: 9 error cos: 0.000264942646 error sin: 0.001129806042
iterations: 10 error cos: 0.0002144575119 error sin: 0.000914812088
iterations: 11 error cos: 0.0001771450043 error sin: 0.0007558465004
iterations: 12 error cos: 0.0001487731934 error sin: 0.0006350874901
iterations: 13 error cos: 0.0001268386841 error sin: 0.0005410313606
iterations: 14 error cos: 0.0001094341278 error sin: 0.0004664063454
iterations: 15 error cos: 9.52482235e-05 error sin: 0.0004062652588
iterations: 16 error cos: 8.374452591e-05 error sin: 0.0003569722176

angle: 2 * pi / 147

iterations: 1 error cos: 1.788139343e-07 error sin: 1.301243901e-05
iterations: 2 error cos: 5.960464478e-08 error sin: 3.252178431e-06
iterations: 3 error cos: 5.960464478e-08 error sin: 1.441687346e-06
iterations: 4 error cos: 0 error sin: 8.121132851e-07
iterations: 5 error cos: 0 error sin: 5.215406418e-07
iterations: 6 error cos: 5.960464478e-08 error sin: 3.61353159e-07
iterations: 7 error cos: 5.960464478e-08 error sin: 2.644956112e-07
iterations: 8 error cos: 5.960464478e-08 error sin: 2.011656761e-07
iterations: 9 error cos: 0 error sin: 1.564621925e-07
iterations: 10 error cos: 5.960464478e-08 error sin: 1.303851604e-07
iterations: 11 error cos: 0 error sin: 1.043081284e-07
iterations: 12 error cos: 5.960464478e-08 error sin: 8.568167686e-08
iterations: 13 error cos: 0 error sin: 7.450580597e-08
iterations: 14 error cos: 0 error sin: 6.332993507e-08
iterations: 15 error cos: 0 error sin: 5.587935448e-08
iterations: 16 error cos: 0 error sin: 4.842877388e-08

angle: 15 * pi / 16

iterations: 1 error cos: 0.03125822544 error sin: 0.1314188838
iterations: 2 error cos: 0.007118344307 error sin: 0.02842205763
iterations: 3 error cos: 0.003113627434 error sin: 0.01232880354
iterations: 4 error cos: 0.001741826534 error sin: 0.006877303123
iterations: 5 error cos: 0.001111924648 error sin: 0.004384636879
iterations: 6 error cos: 0.0007711052895 error sin: 0.003038585186
iterations: 7 error cos: 0.0005660653114 error sin: 0.002229571342
iterations: 8 error cos: 0.0004332065582 error sin: 0.001705765724
iterations: 9 error cos: 0.0003421902657 error sin: 0.001346945763
iterations: 10 error cos: 0.0002770423889 error sin: 0.001090586185
iterations: 11 error cos: 0.000228944406 error sin: 0.0009011030197
iterations: 12 error cos: 0.0001923441887 error sin: 0.000756919384
iterations: 13 error cos: 0.0001638531685 error sin: 0.0006448030472
iterations: 14 error cos: 0.0001413226128 error sin: 0.0005558729172
iterations: 15 error cos: 0.0001232028008 error sin: 0.0004842281342
iterations: 16 error cos: 0.0001082420349 error sin: 0.0004254579544

angle: 3 * pi / 147

iterations: 1 error cos: 6.556510925e-07 error sin: 4.391372204e-05
iterations: 2 error cos: 1.192092896e-07 error sin: 1.096725464e-05
iterations: 3 error cos: 5.960464478e-08 error sin: 4.87267971e-06
iterations: 4 error cos: 5.960464478e-08 error sin: 2.74181366e-06
iterations: 5 error cos: 5.960464478e-08 error sin: 1.75088644e-06
iterations: 6 error cos: 0 error sin: 1.214444637e-06
iterations: 7 error cos: 5.960464478e-08 error sin: 8.940696716e-07
iterations: 8 error cos: 5.960464478e-08 error sin: 6.854534149e-07
iterations: 9 error cos: 0 error sin: 4.438923836e-07
iterations: 10 error cos: 0 error sin: 3.321336746e-07
iterations: 11 error cos: 0 error sin: 3.576278687e-07
iterations: 12 error cos: 5.960464478e-08 error sin: 2.980232239e-07
iterations: 13 error cos: 5.960464478e-08 error sin: 2.682209015e-07
iterations: 14 error cos: 0 error sin: 2.235174179e-07
iterations: 15 error cos: 5.960464478e-08 error sin: 1.937150955e-07
iterations: 16 error cos: 0 error sin: 1.713633537e-07

angle: 16 * pi / 16

iterations: 1 error cos: 0.04030227661 error sin: 0.1585290432
iterations: 2 error cos: 0.009052276611 error sin: 0.0335290432
iterations: 3 error cos: 0.003951132298 error sin: 0.01449614763
iterations: 4 error cos: 0.002208709717 error sin: 0.008077383041
iterations: 5 error cos: 0.001409590244 error sin: 0.0051471591
iterations: 6 error cos: 0.000972777557 error sin: 0.003566026688
iterations: 7 error cos: 0.0007173418999 error sin: 0.002616286278
iterations: 8 error cos: 0.0005488991737 error sin: 0.002001285553
iterations: 9 error cos: 0.0004335045815 error sin: 0.001580238342
iterations: 10 error cos: 0.0003510117531 error sin: 0.0012794137
iterations: 11 error cos: 0.0002909958061 error sin: 0.001057088375
iterations: 12 error cos: 0.0002437829971 error sin: 0.0008879899979
iterations: 13 error cos: 0.0002076029778 error sin: 0.0007564425468
iterations: 14 error cos: 0.000178873539 error sin: 0.0006522536278
iterations: 15 error cos: 0.0001559257507 error sin: 0.0005682110786
iterations: 16 error cos: 0.000137090683 error sin: 0.0004991889

angle: 4 * pi / 147

iterations: 1 error cos: 2.205371857e-06 error sin: 0.0001040771604
iterations: 2 error cos: 5.36441803e-07 error sin: 2.598762512e-05
iterations: 3 error cos: 2.384185791e-07 error sin: 1.154094934e-05
iterations: 4 error cos: 1.192092896e-07 error sin: 6.4894557e-06
iterations: 5 error cos: 5.960464478e-08 error sin: 4.157423973e-06
iterations: 6 error cos: 1.192092896e-07 error sin: 2.883374691e-06
iterations: 7 error cos: 5.960464478e-08 error sin: 2.11596489e-06
iterations: 8 error cos: 5.960464478e-08 error sin: 1.61677599e-06
iterations: 9 error cos: 0 error sin: 1.281499863e-06
iterations: 10 error cos: 0 error sin: 1.043081284e-06
iterations: 11 error cos: 1.192092896e-07 error sin: 8.642673492e-07
iterations: 12 error cos: 0 error sin: 7.301568985e-07
iterations: 13 error cos: 5.960464478e-08 error sin: 6.034970284e-07
iterations: 14 error cos: 5.960464478e-08 error sin: 5.289912224e-07
iterations: 15 error cos: 0 error sin: 4.470348358e-07
iterations: 16 error cos: 5.960464478e-08 error sin: 3.87430191e-07

.....

angle: 32 * pi / 147

iterations: 1 error cos: 0.008973300457 error sin: 0.05207568407
iterations: 2 error cos: 0.00213766098 error sin: 0.01209431887
iterations: 3 error cos: 0.0009419322014 error sin: 0.005305945873
iterations: 4 error cos: 0.0005282759666 error sin: 0.002971231937
iterations: 5 error cos: 0.0003376603127 error sin: 0.00189769268
iterations: 6 error cos: 0.0002343654633 error sin: 0.00131636858
iterations: 7 error cos: 0.0001720190048 error sin: 0.000966489315
iterations: 8 error cos: 0.0001316666603 error sin: 0.0007395744324
iterations: 9 error cos: 0.0001040101051 error sin: 0.0005843043327
iterations: 10 error cos: 8.428096771e-05 error sin: 0.0004731416702
iterations: 11 error cos: 6.955862045e-05 error sin: 0.0003908872604
iterations: 12 error cos: 5.853176117e-05 error sin: 0.000328540802
iterations: 13 error cos: 4.976987839e-05 error sin: 0.0002798438072
iterations: 14 error cos: 4.297494888e-05 error sin: 0.0002413392067
iterations: 15 error cos: 3.743171692e-05 error sin: 0.0002101659775
iterations: 16 error cos: 3.296136856e-05 error sin: 0.0001847147942

from 1 to 32 largest error: 0.05207568407

test angle: $(1 \text{ to } 997) * (\pi/2) / 997$

largest error at 1 iterations: 0.570796
largest error at 2 iterations: 0.0863233
largest error at 3 iterations: 0.0359604
largest error at 4 iterations: 0.0197986
largest error at 5 iterations: 0.0125484
largest error at 6 iterations: 0.00866878
largest error at 7 iterations: 0.00634885
largest error at 8 iterations: 0.0048511
largest error at 9 iterations: 0.00382769
largest error at 10 iterations: 0.00309741
largest error at 11 iterations: 0.00255775
largest error at 12 iterations: 0.00214815
largest error at 13 iterations: 0.0018295
largest error at 14 iterations: 0.00157702
largest error at 15 iterations: 0.00137329
largest error at 16 iterations: 0.00120676

average error at 1 iterations: cos:	0.0479706	sin:	0.149065
average error at 2 iterations: cos:	0.00982466	sin:	0.0277034
average error at 3 iterations: cos:	0.00423763	sin:	0.0118079
average error at 4 iterations: cos:	0.00235966	sin:	0.00654944
average error at 5 iterations: cos:	0.00150321	sin:	0.00416494
average error at 6 iterations: cos:	0.00104129	sin:	0.00288237
average error at 7 iterations: cos:	0.000763884	sin:	0.00211329
average error at 8 iterations: cos:	0.00058428	sin:	0.00161582
average error at 9 iterations: cos:	0.000461346	sin:	0.00127553
average error at 10 iterations: cos:	0.000373513	sin:	0.0010325
average error at 11 iterations: cos:	0.000308582	sin:	0.000852893
average error at 12 iterations: cos:	0.000259223	sin:	0.000716403
average error at 13 iterations: cos:	0.000220833	sin:	0.000610252
average error at 14 iterations: cos:	0.000190381	sin:	0.000526069
average error at 15 iterations: cos:	0.000165821	sin:	0.000458181
average error at 16 iterations: cos:	0.000145729	sin:	0.000402638

997 tests to $\pi/4$:

average error at 1 iterations: cos:	0.00313249	sin:	0.0198151
average error at 2 iterations: cos:	0.000748374	sin:	0.00464497
average error at 3 iterations: cos:	0.000329948	sin:	0.00204126
average error at 4 iterations: cos:	0.00018508	sin:	0.00114373
average error at 5 iterations: cos:	0.000118299	sin:	0.000730673
average error at 6 iterations: cos:	8.20971e-05	sin:	0.000506914
average error at 7 iterations: cos:	6.02907e-05	sin:	0.000372211
average error at 8 iterations: cos:	4.61501e-05	sin:	0.000284865
average error at 9 iterations: cos:	3.64571e-05	sin:	0.00022502
average error at 10 iterations: cos:	2.95286e-05	sin:	0.000182232
average error at 11 iterations: cos:	2.4402e-05	sin:	0.000150585
average error at 12 iterations: cos:	2.05043e-05	sin:	0.000126519
average error at 13 iterations: cos:	1.74732e-05	sin:	0.000107794
average error at 14 iterations: cos:	1.5066e-05	sin:	9.29386e-05
average error at 15 iterations: cos:	1.31229e-05	sin:	8.09564e-05
average error at 16 iterations: cos:	1.15369e-05	sin:	7.11499e-05

improved accuracy at smaller angles :)